INTERNATIONAL JOURNAL OF ADVANCED ENGINEERING AND BUSINESS SCIENCES (IJAEBS)

Journal homepage: https://ijaebs.journals.ekb.eg

NON-LINEAR EFFECT ON FLEXURAL STIFFNESS OF REINFORCED CONCRETE RECTANGULAR SECTIONS

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Received: 28-01-2024 **Accepted:** 29-2-2024 **Published**: 01-03-2024

ABSTRACT

The load-deformation characteristics of flexural members are very important in designing reinforced concrete structures. In most cases the dimensions of structural elements are governed by the allowable deflections to comply with the serviceability limit state provisions. Flexural rigidity represents a governing factor in the member deflection calculation. Due to cracking and the non-linear behavior of concrete, the flexural rigidity changes during loading history. Most current Codes recommend using the concept of equivalent moment of inertia and concrete elastic modulus for calculating immediate deflections. The aims of this research are: investigating the load-deformation characteristics taking into account the non-linear behavior of concrete; and introducing a simple method for estimating the flexural rigidity of reinforced concrete rectangular sections. A parametric study is carried out on groups of rectangular reinforced concrete sections for checking its flexural rigidity. Using regression analysis on the results of parametric study, empirical formula for estimating flexural rigidity is obtained. Also, a mathematical approach and design-aids charts are prepared for evaluating the flexural rigidity. Comparison of the results obtained using the proposed techniques with the nonlinear analysis results shows good agreement.

Keywords- flexural rigidity; moment-curvature; ductile failure; non-linear behavior; mathematical approach.

1 INTRODUCTION

The immediate deflections of structural members depend on the flexural rigidity (EI). In reinforced concrete members, cracks have a significant influence on the value of the moment of inertia (I). Before cracking, the flexural rigidity can be evaluated using concrete elastic modulus and the moment of inertia of the gross uncracked section. For bending moments great enough to produce tensile stresses greater than the modulus of rupture, cracks will form reducing the moment of inertia (Icr). Between cracks, concrete carry some tension, because tension is transformed from steel to concrete by bond and a sufficient length is required for the tensile stress in concrete to reach the modulus of rupture before cracking again. The tension carried by concrete between cracks will tend to stiffen the member [1] [2]. Also, in the regions of small bending moments, the member is still uncracked. The present research aims to estimate the flexural rigidity of rectangular sections for different load levels taking into account cracking and concrete non-linear behavior. The load-deformation characteristics of flexural members will be examined. Such characteristics are mainly dependent on the moment-curvature characteristics of the sections, since most of the deformations of members of normal proportions arise from strains associated with flexure.

2 CODE METHOD

The Egyptian Code for the Design and Construction of Concrete Structures (ECP-203) [3] and the ACI 318-89 [4] use the concept of equivalent moment of inertia and concrete elastic modulus for calculating immediate deflections [5] [6]. The equivalent moment of inertia can be estimated according to the following equation:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \tag{1}$$

where Ig is the moment of inertia of the gross uncracked section, Icr is the moment of inertia of the cracked section, Ma is the applied moment at stage at which the deflection is required, and Mcr is the cracking moment, given by

$$M_{cr} = \frac{f_{ctr}\ I_g}{y_t} \tag{2}$$

where fctr is the modulus of rupture of concrete, and yt is the distance between the centroidal axis of gross section and the fiber of maximum tensile stress [7] [8]. The ECP defines the concrete elastic modulus to be:

$$E_c = 4400\ \sqrt{f_{cu}} \tag{3}$$

where f_{cu} is the concrete compressive characteristic strength (N/mm^2).

3 MOMENT-CURVATURE RELATIONSHIPS

The relationship between moment M and curvature φ (the rotation per unit length) is given by the following classical elastic equation [9] [10]:

$$EI = \frac{M}{\varphi} \tag{4}$$

For a small element of length *dx* of the member, the curvature can be given by:

$$\varphi = \frac{\varepsilon_c}{c} = \frac{\varepsilon_c + \varepsilon_s}{d} \tag{5}$$

where ε_c is concrete strain at the extreme compression fiber, ε_s is the strain in tension steel, *c* is the neutral axis depth, and *d* is the depth of tension steel. The behavior of the section after cracking depends mainly on the steel content.

The theoretical moment-curvature curves for reinforced concrete sections are determined considering the following assumptions [11] [12]: plane sections before bending remain plane after bending; and the stress-strain curves of concrete and steel are taken as the idealized curves recommended by ECP. The stress-strain curve of concrete in compression is idealized by a parabolic curve up to a strain of 0.002 and a straight horizontal line of stress 0.67fcu up to a strain of 0.003. The parabolic curve and the modulus of rupture of concrete are expressed by:

$$f_c = 0.67\ f_{cu} \left[\frac{2\,\varepsilon_c}{0.002} - \left(\frac{\varepsilon_c}{0.002} \right)^2 \right] \qquad (N/mm^2) \tag{6}$$

$$f_{ctr} = 0.6\sqrt{f_{cu}} \qquad (N/mm^2) \tag{7}$$

The stress-strain curve of steel, in both tension and compression, is represented by an elastoplastic curve with elastic modulus E_s = 200000 N/mm^2 up to the yield stress.

A program is designed to calculate moments and curvatures for rectangular sections using iterative technique [13]. First, the cracking moment is determined by assuming the strain at the extreme tension fiber equals cracking strain. Then, the section is divided into strips and the neutral axis depth is assumed as shown in Fig. 1 [14] [15]. The strain at each strip is determined according to the distance from the neutral axis. The stresses in each strip are calculated using the suggested stress-strain curves. The tensile stresses in the concrete fibers are taken into account. The total tensile and compressive forces are determined. The equilibrium between tensile and compressive forces is checked, and then the neutral axis depth is modified until equilibrium. At equilibrium, the internal moment and the associated curvature are calculated.

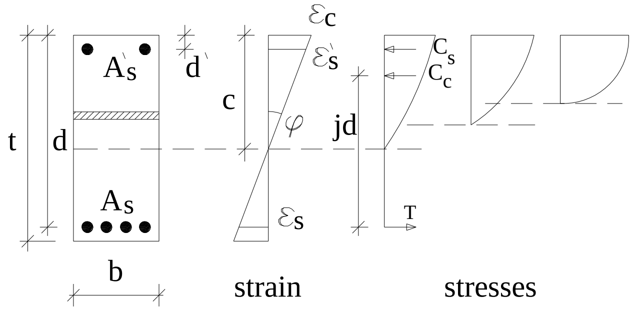


Figure 1. *Example of the analyzed sections.*

Second, to change form the uncracked to cracked section the movement of the neutral axis depth is checked gradually. The strain in the tension steel is increased by a small amount and again the strains at each strip and the total tensile and compressive forces are determined according to an assumed neutral axis depth. The process is repeated until equilibrium. At each step, the stresses in the strips at the tension zone are checked to take into account the tensile stresses in the uncracked strips directly below the neutral axis. Third, the process is continued until the strain at tension steel reaches the yield strain. Similarly, the yielding moment and the associated curvature are determined. Finally, in the same manner, the ultimate moment and curvature are estimated.

3.1 Investigated Parameters

The program is used to obtain the moment-curvature relationships for different rectangular sections. In the present research, the investigated parameters are the concrete compressive characteristic strength (f_{cu}), the yield stress of reinforcing steel (f_y), the reinforcement ratio (μ), the ratio of compression steel (α), and the ratio between the depths of both compression and tension steel (d'/d).

The concrete compressive characteristic strength is taken 25, 30, 35, 40, and 45 N/mm^2, while the reinforcement yield stress is taken as 240, 280, 360, and 400 N/mm^2. The reinforcement ratio varies between the minimum and maximum values according to the requirements of the ECP. The ratio of compression steel is taken 0.1, 0.2, 0.3, and 0.4. The ratio (d'/d) is varied according to the section dimension and the volume of reinforcing steel in tension and compression.

4 ANALYSIS OF RESULTS

4.1 Schematic Moment-Curvature Curve

More than six hundred sections are analyzed by the program and the results are obtained and represented to evaluate the effect of the investigated parameters. The moment-curvature curve for each case is represented by more than four hundred points. Figure 2 represents a schematic curve of the obtained moment-curvature relationships. From the figure, it can be noticed that the curve starts with a straight line with an initial flexural rigidity up to the point of first cracking (cracking point). Once cracks have developed, the initial flexural rigidity disappears and a sharply inclined small straight line represents the relationship. This line is inclined and not horizontal. The line represents moving the neutral axis from uncracked to cracked section. The first point is the first cracking point, i.e. at M_{cr}. The second point of that line has larger moment and curvature because the parts between the extreme tension fiber and the neutral axis are still able to carry some tensile stresses until reaching its tensile strength. This line may be titled as *crack propagation line*.

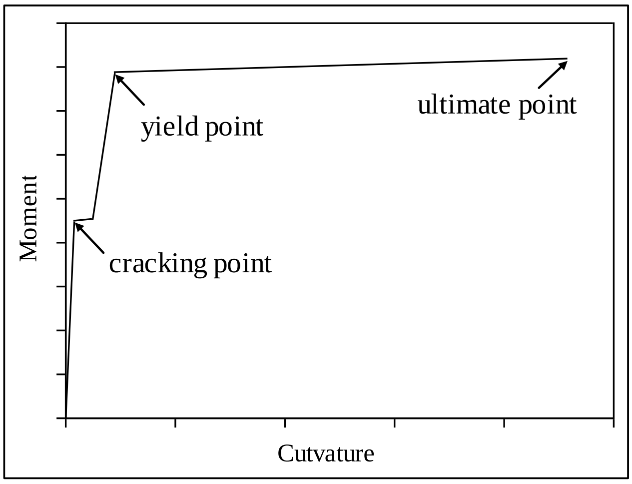
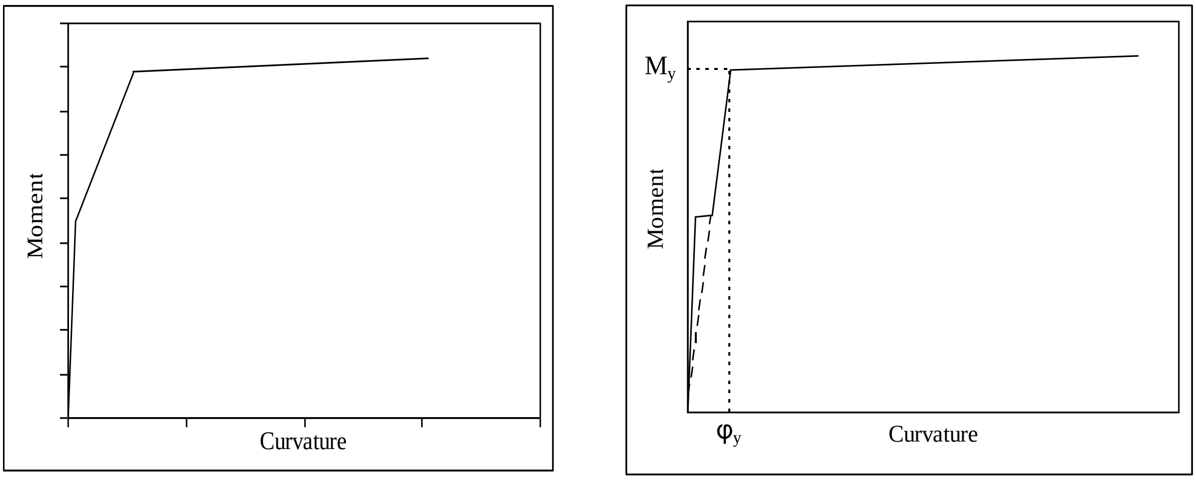


Figure 2. ***Schematic curve for the obtained moment-curvature relationship.***

For the cracked section, the moment-curvature relationship is represented by approximately straight line up to the yielding point [16]. Yielding point represents the moment and curvature when the tension steel reaches to yield. After yielding, it can be noticed a small increase in moment accompanied with large increase in curvature up to ultimate point. Some texts and researches represent the moment-curvature relationship by trilinear relationship as shown in Fig. 3-a. The first part is up to cracking, the second to yield of the tension steel, and the third to the ultimate point. This relationship neglects the stage of moving the neutral axis from uncraked case to cracked one. Figure 3-b explains that the part after cracking to yielding point have approximately the same flexural rigidity. This behavior cannot be noticed in the classical relationship. From Fig. 3-b, the flexural rigidity of the section after cracking and up to yielding can be expressed as:

$$EI = \frac{M_y}{\varphi_y} \tag{8}$$



(a) *Classical moment-curvature relationship.* (b) *Proposed moment-curvature relationship.*

Figure 3. ***Moment-curvature relationships.***

The previous remarks are noticed for all the obtained curves for different parameters. Figure 4 shows schematic curves for different reinforcing ratios, which increase from μ_1 to μ_4.

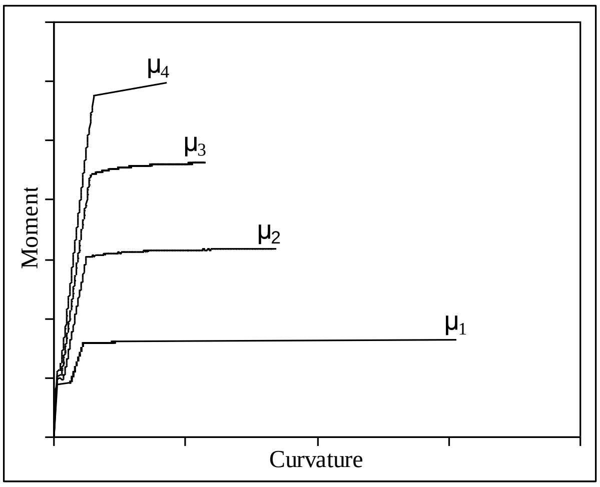
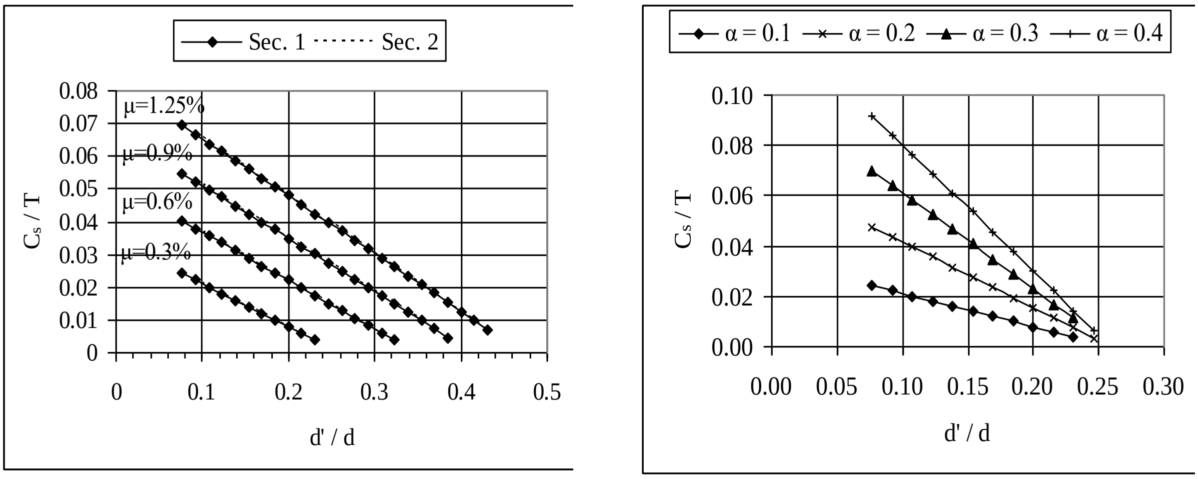


Figure 4. ***Schematic moment-curvature curves for different reinforcing ratios.***

4.2 Internal Forces

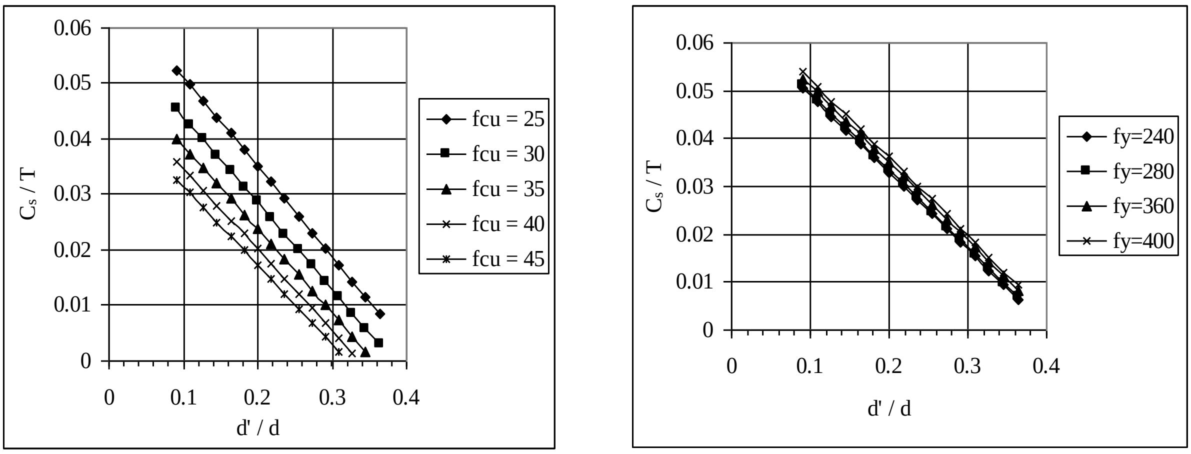
For each case study, the internal forces are examined. Figure 5-a represents the internal forces in compression steel (C_s) for f_{cu} = 25 N/mm^2, f_y = 360 N/mm^2 and compression steel ratio α = 0.1. The figure represents the internal forces in compression steel at yielding as a ratio from the tension steel forces (T) for different ratios (d'/d) and different reinforcing ratios (μ). In addition, the figure represents the obtained results for groups of two different cross-sections (different b and t). From this figure, it can be noticed that, first, the same outputs are obtained for different cross sections.

Second, the behavior of the sections is the same for different reinforcing ratio, i.e., linear with approximately the same slop, but with different constants of relation. The same remarks are obtained for different values of α. Figure 5-b represents the effect of compression steel ratio on the internal force (C_s). It can be concluded that the slope of the line depends on the compression steel content. The same results are obtained for different concrete compressive characteristic strength. The effects of concrete strength and yield stress are represented in Fig. 6. As noticed from Fig. 6-a, the constant of the linear relationships depends non-linearly on the concrete strength. Figure 6-b represents the slight effect of the yield stress on the ratio (C_s/T). From this figure, it can be noticed that increasing the yield stress from 240 N/mm^2 to 400 N/mm^2 increases the ratio C_s/T by about 0.003.



(a) $\alpha = 0.1$, $f_{cu} = 25$ N/mm^2, and $f_y = 360$ N/mm^2 (b) $\mu = 0.3\%$, $f_{cu} = 25$ N/mm^2, and $f_y = 360$ N/mm^2

Figure 5. ***Internal forces in compression steel.***



(a) $\alpha = 0.1$, $\mu = 0.9\%$, and $f_y = 360$ N/mm^2 (b) $\alpha = 0.1$, $\mu = 0.9\%$, and $f_{cu} = 25$ N/mm^2

Figure 6. ***Internal forces in compression steel.***

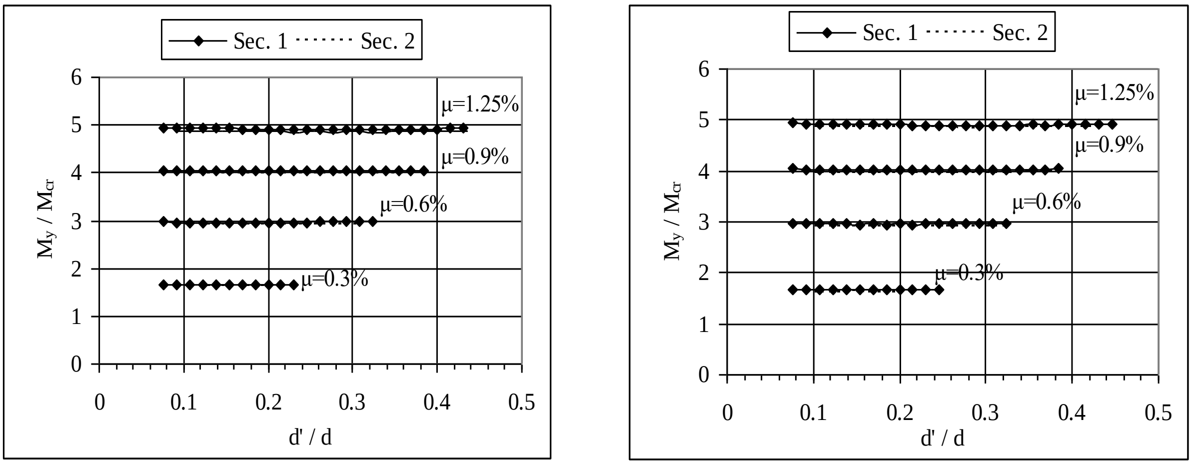
The previous observations can be explained mathematically as following:

$$C_s = E_s\ \varepsilon_s'\ A_s' = E_s\ A_s'\ \frac{\varepsilon_y\ (c-d')}{(d-c)} = E_s\ A_s'\ \frac{\varepsilon_y\ ((c/d)-(d'/d))}{(1-(c/d))} \tag{9}$$

$$\frac{C_s}{T} = \frac{E_s\ A_s'}{E_s\ \varepsilon_y\ A_s} \times \frac{\varepsilon_y\ ((c/d)-(d'/d))}{(1-(c/d))} = \frac{\alpha\ ((c/d)-(d'/d))}{(1-(c/d))} \tag{10}$$

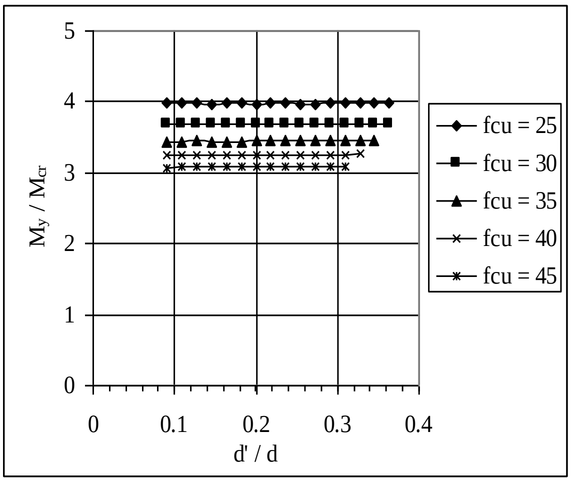
As shown in Eqs. (9) and (10), the ratio (C_s/T) depends on the compression steel ratio (α), the ratio of the neutral axis depth to the section depth (c/d), and the depth ratio of both compression and tension steel (d'/d). In addition, the force in both compression steel (C_s), concrete in compression zone (C_c), and tension steel (T) are balanced. These forces depend on concrete strength and reinforcing ratio.

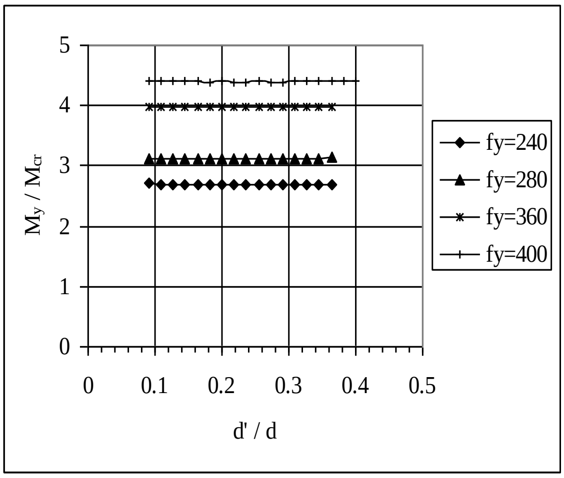
The moment of internal forces, at yielding point, are obtained, and the ratios of yielding moments and cracking moments, calculated by using Eq. (2), are determined. Figure 7 represents the ratio (M_y/M_{cr}) for α = 0.1 and 0.2. From these figures, it can be noticed that the ratio (M_y/M_{cr}) can be considered constant for the same reinforcing ratio and different contents of compression steel. The same results are obtained for the cases of α = 0.3 and 0.4. The effect of concrete strength and yield stress on the moment ratio is illustrated in Fig. 8. From this figure it can be noticed that, concrete strength affects non-linearly the moment ratio (M_y/M_{cr}), while the ratio is related linearly to the yield stress value.



(a) $\alpha = 0.1$ (b) $\alpha = 0.2$

Figure 7. ***Yielding moments (f_{cu} = 25 N/mm^2 and f_y = 360 N/mm^2).***





(a) $\alpha = 0.1$, $\mu = 0.9\%$, and $f_y = 360\ N/mm^2$ (b) $\alpha = 0.1$, $\mu = 0.9\%$, and $f_{cu} = 25\ N/mm^2$

Figure 8. ***Yielding moments.***

For the studied cases, the curvatures at both cracking and yielding are calculated according to Eq. (5). The ratio of yielding and cracking curvatures (φ_y/φ_{cr}) are compared for different sections and reinforcing ratios as shown in Fig. 9. Due to the small values of curvatures and using crude values (not approximated), a slight difference can be noticed for the outputs of different sections. The same curvature ratios for different values of μ are obtained for the other compression steel contents (α = 0.2, 0.3 and 0.4). The previous behaviors are noticed for different concrete strengths, as illustrated in Fig. 10-a. From this figure, it can be noticed that increasing the concrete strength from 25 to 45 N/mm^2 decreases the curvature ratio from about 11.5 to 10, which can be considered as a considerably slight effect. This effect is more obviously expressed in Fig. 10-b. The curvature ratios for different steel type are illustrated in Fig. 11. The curvature ratio is related linearly to the yield stress value.

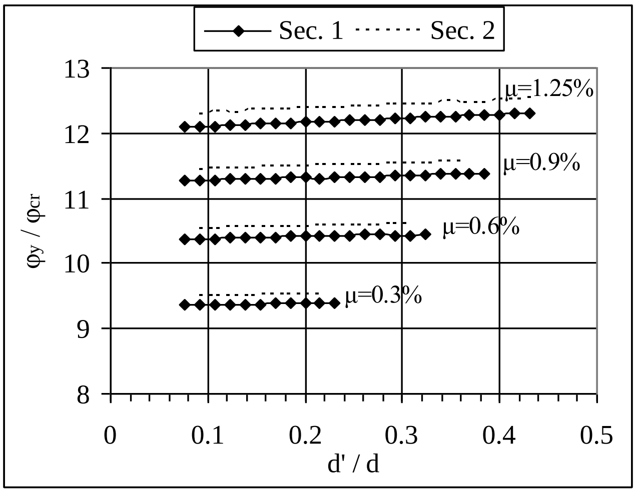
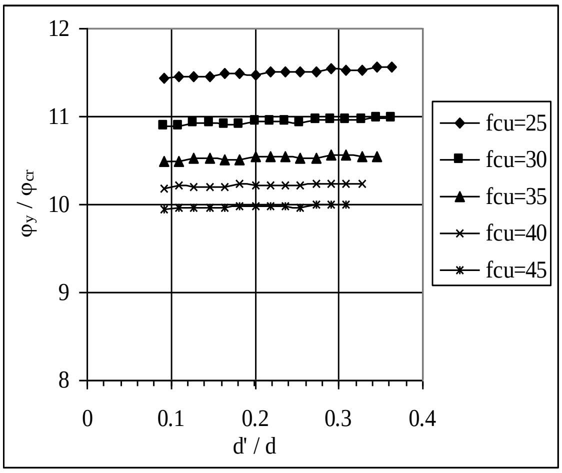
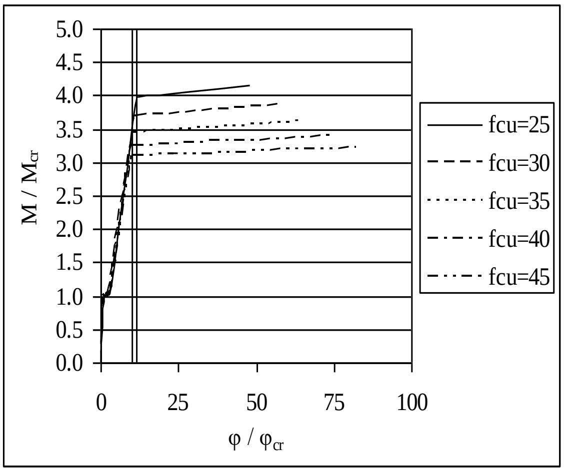


Figure 9. ***Curvature ratio ($\alpha = 0.1$, $f_{cu} = 25\ N/mm^2$, and $f_y = 360\ N/mm^2$).***





(a) **(b)**

Figure 10. ***Curvature ratio (α = 0.1, μ = 0.9 %, and f_y = 360 N/mm^2).***

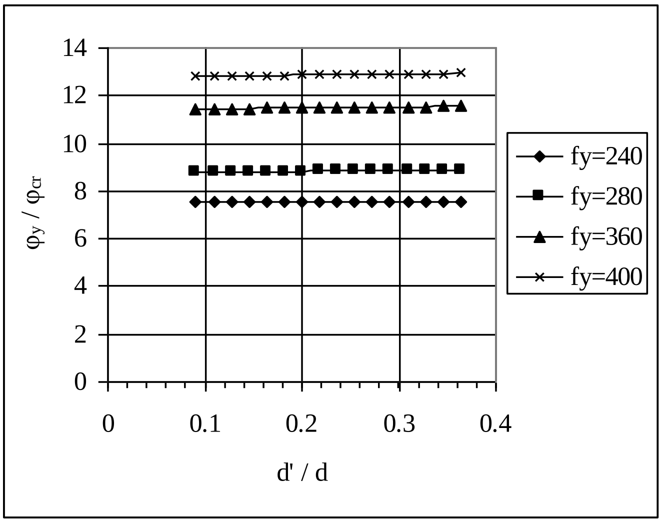
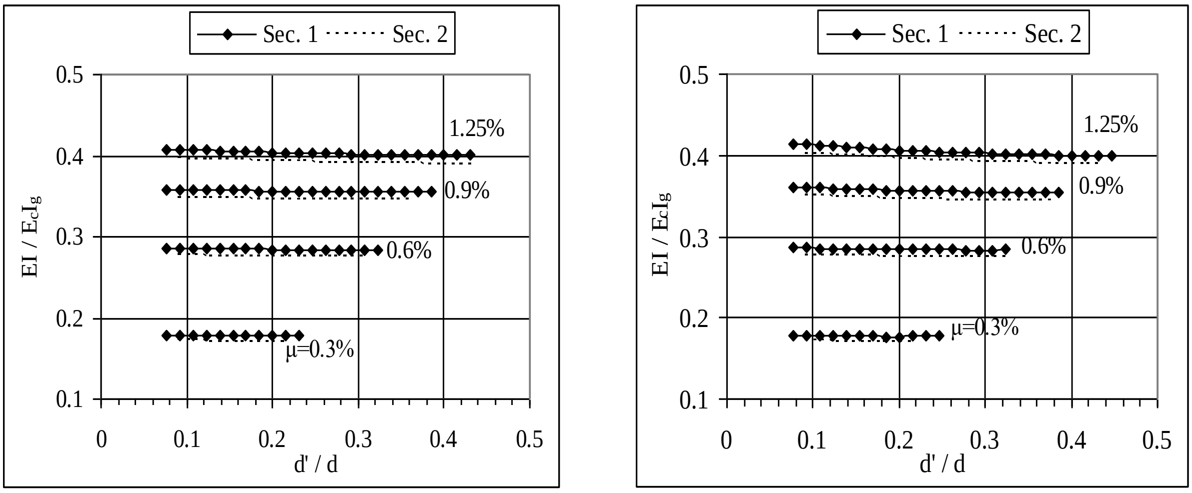


Figure 11. ***Curvature ratio (α = 0.1, μ = 0.9 %, and f_{cu} = 25 N/mm^2).***

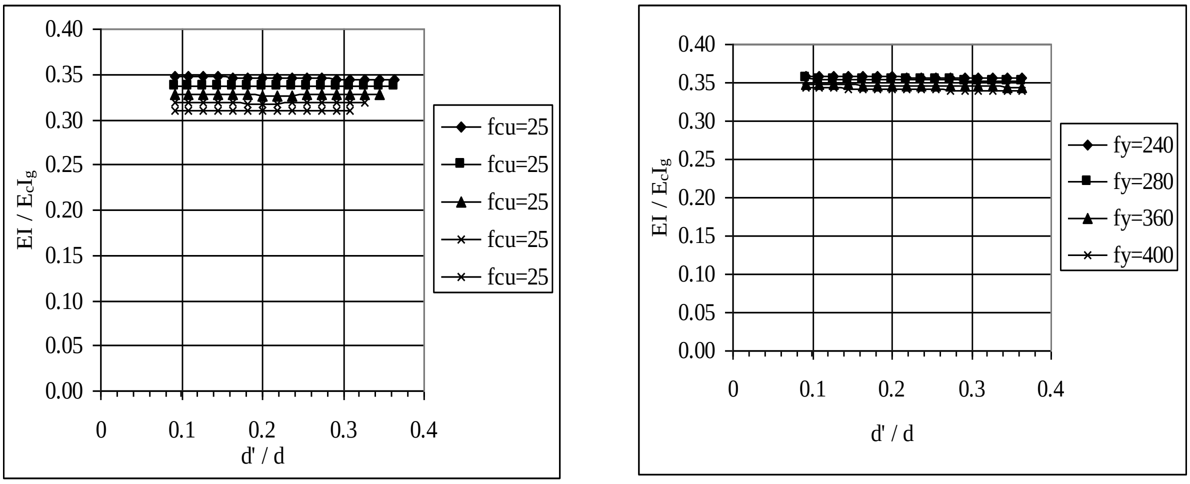
The flexural rigidity of the studied sections is calculated for the loading part from cracking to yielding (the line shown in Fig. 3-b) as M_y/φ_y. Figure 12 represents the ratio of the calculated flexural rigidity and the flexural rigidity of uncracked section (EI/E_cI_g). Approximately, the same ratio can be noticed for the same reinforcing ratio and different compression steel content. The same observations are found for both α = 0.3 and 0.4. Figure 13 explains the effect of concrete strength and yield stress on the flexural rigidity ratio. As shown in Fig. 13-a, increasing the concrete strength from 25 to 45 N/mm^2 decreases the ratio by about 0.04. For different steel types, the ratio of flexural rigidity ranges around 0.35 for yield stresses from 240 to 400 N/mm^2, as noticed from Fig. 13-b.

From these figures it can be said that, for reinforced concrete rectangular sections with different dimensions the flexural rigidity ratio depends mainly on the concrete compressive characteristic strength and the reinforcing ratio.



(a) ***α = 0.1*** **(b)** ***α = 0.2***

Figure 12. ***Ratio of flexural rigidity (f_{cu} = 25 N/mm^2 and f_y = 360 N/mm^2).***



(a) ***(α = 0.1, μ = 0.9 %, and f_y = 360 N/mm^2)*** **(b)** ***(α = 0.1, μ = 0.9 %, and f_{cu} = 25 N/mm^2)***

Figure 13. ***Ratio of flexural rigidity.***

5 REGRESSION ANALYSIS

In this part, the least squares method and the multiple regression technique are used as powerful tools in achieving relationships between variables [17] [18]. Investigating the obtained results, it can be found that the flexural rigidity ratio is linearly related to the concrete strength, but non-linearly to the reinforcing ratio. The flexural rigidity ratios for different concrete strengths are shifted equally. Taking fcu = 25 N/mm2 as a datum (since it is practically the minimum strength for reinforced concrete members), it is found that the shift between the flexural rigidity ratios (from the datum value) for different concrete strengths is:

$$\Delta = 0.05\left(\left(f_{cu}\ /\ 25\right) - 1\right) \tag{11}$$

where Δ is the shift of the flexural rigidity ratio, and f_{cu} is expressed in N/mm^2. For the effect of reinforcing ratio, it is found that the relation can be well expressed in a natural logarithm form. The proposed empirical formula for estimating the ratio of the flexural rigidity of a rectangular section with concrete strength f_{cu} (expressed in N/mm^2) and a reinforcing ratio μ (%) is:

$$\frac{EI}{E_c I_g} = 0.17\ \ln\left(\mu\right) - \Delta + 0.37 \tag{12}$$

6 MATHEMATICAL APPROACH

Another technique depends on evaluating the ratio (C_c/T), then using assumptions and equilibrium equations; the flexural rigidity can be estimated. The ratio (C_c/T) can be determined as following:

$$\frac{C_c}{T} = 1 - \frac{C_s}{T} = 1 - \frac{\alpha\left(\left(c/d\right) - \left(d'/d\right)\right)}{\left(1 - \left(c/d\right)\right)} \tag{13}$$

According to the assumed concrete stress-strain curve, the compressive force in concrete can be determined as:

$$C_c = \left[\int_0^c 0.67\ f_{cu}\left(\frac{2\,\varepsilon_{c_i}}{0.002} - \left(\frac{\varepsilon_{c_i}}{0.002}\right)^2\right) dc_i\right] \times b \tag{14}$$

where, $$\varepsilon_c = \frac{\varepsilon_y\left(c/d\right)}{\left(1 - c/d\right)}\ ,\qquad \varepsilon_{c_i} = \frac{\varepsilon_c\ c_i}{c} \tag{15}$$

Then, $$C_c = \left[\int_0^c 0.67\ f_{cu}\left(\frac{2\,\varepsilon_c\ c_i}{0.002\,c} - \left(\frac{\varepsilon_c\ c_i}{0.002\,c}\right)^2\right) dc_i\right] \times b \tag{16}$$

$$C_c = 0.67\ f_{cu}\ b\left(\frac{\varepsilon_c\ c}{0.002} - \frac{c}{3}\left(\frac{\varepsilon_c}{0.002}\right)^2\right) = 0.67\ f_{cu}\ c\ b\left(\frac{\varepsilon_c}{0.002} - \frac{1}{3}\left(\frac{\varepsilon_c}{0.002}\right)^2\right) \tag{17}$$

$$\frac{C_c}{T} = \frac{0.67\ f_{cu}\ c\ b\ d}{E_s\ \varepsilon_y\ A_s\ d}\left(\frac{\varepsilon_c}{0.002} - \frac{1}{3}\left(\frac{\varepsilon_c}{0.002}\right)^2\right) \tag{18}$$

$$\frac{C_c}{T} = \frac{0.67\ f_{cu}}{f_y\ \mu}\left(\frac{\varepsilon_c}{0.002} - \frac{1}{3}\left(\frac{\varepsilon_c}{0.002}\right)^2\right)\frac{c}{d} \tag{19}$$

The ratio (C_c/T) obtained by both Eqs. (13) and (19) must be the same, i.e., it is required now to get the ratio (c/d) that makes the difference between the outputs of Eqs. (13) and (19) tends to zero. Figure 14 shows a schematic diagram for the curves of both Eqs. (13) and (19) and the required intersection point. A group of charts for different values of yield stress and compressive strength can be prepared for using in codes. Figure 15 represents a sample of (*c/d*) chart for 360 N/mm^2 yield stress, 25 N/mm^2 concrete compressive strength and different values of reinforcing ratio and compression steel content. The chart can be used for obtaining estimation for the ratio (*c/d*). For higher degrees of accuracy, Eqs. (13) and (19) can be used to increase the accuracy of the obtained ratio. Then, using Eq. (10) the value C_s can be determined and the yield moment and the yield curvature can be estimated as following:

$$M_y = C_s \left(\frac{3c}{8} - d' \right) + T \left(d - \frac{3c}{8} \right) \tag{20}$$

$$\varphi_y = \frac{\varepsilon_y}{(d - c)} \tag{21}$$

Knowing the yield moment and curvature, the flexural rigidity can be estimated from Eq. (8).

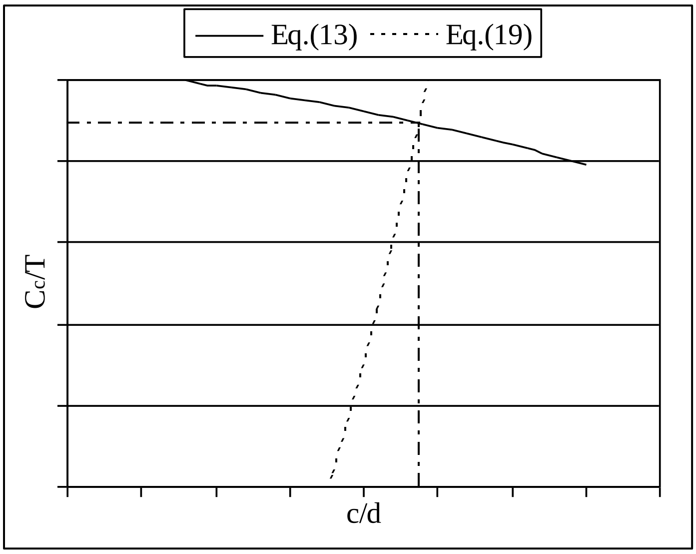


Figure 14. ***Schematic diagram for Eqs. (13) and (19).***

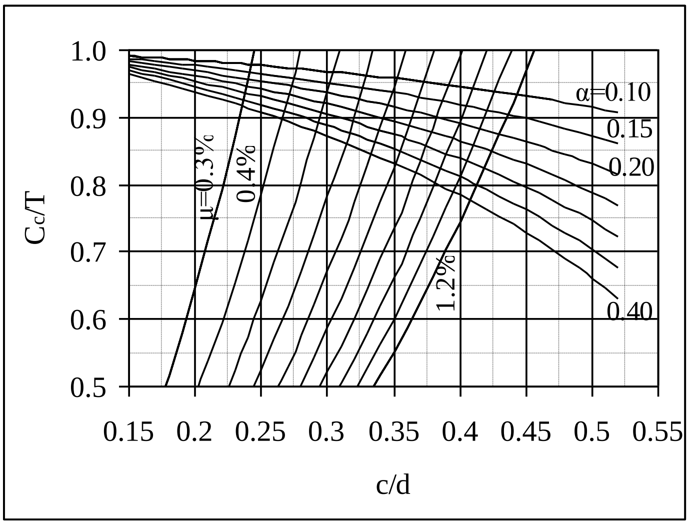
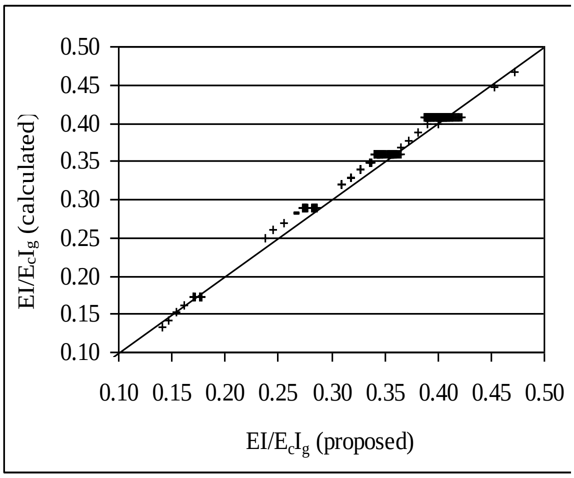


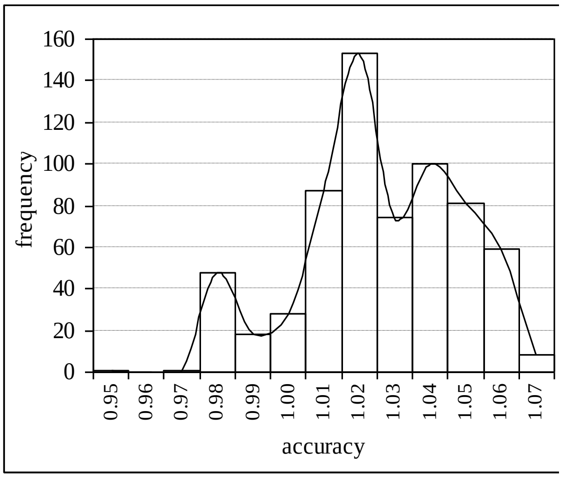
Figure 15. ***Chart of (c/d) (f_{cu} = 25 N/mm^2 and f_y = 360 N/mm^2).***

7 EVALUATING PROPOSED APPROACHES

To asses the accuracy of the proposed formula and the suggested approach, a verification study is carried-out. The flexural rigidity of the investigated sections, in the parametric study, is calculated using the proposed formulae and the obtained results are compared with that obtained by the iterative method. Figure 16-a represents the ratio of the flexural rigidity obtained by using Eq. (12) and the calculated ones (by iterative method). From the figure, it can be noticed that the results are very close to the equality line. Investigating the results it is found that, the ratio of the proposed results and the calculated ones ranges from 0.94 to 1.06 with 1.02 average value, 1.02 median value, 1.01 mode value, 0.02 standard deviation, and 2% coefficient of variance. Figure 16-b represents the histogram of the proposed formula results.



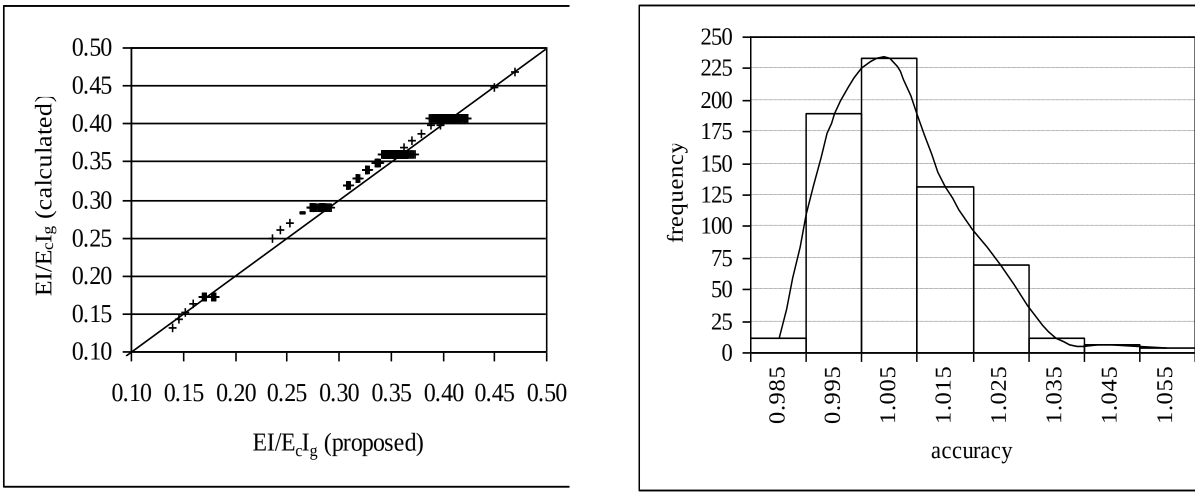
(a) ***Proposed and calculated flexural rigidities.***



(b) ***Histogram of the results of the proposed formula.***

Figure 16. ***Results of flexural rigidity proposed formula.***

The flexural rigidities of the studied sections are estimated using the proposed mathematical approach. Figure 17-a represents the ratio of the flexural rigidity obtained by using the mathematical approach and the calculated ones. From the figure, it can be noticed that the results are very close to the equality line. Investigating the results it is found that, the ratio of the proposed results and the calculated ones ranges from 0.99 to 1.04 with 1.007 average value, 1.006 median value, 1.005 mode value, 0.004 standard deviation, and 0.4% coefficient of variance. Figure 17-b represents the histogram of the proposed formula results.



(a) ***Proposed and calculated flexural rigidities.***

(b) ***Histogram of the results of the proposed mathematical approach.***

Figure 17. ***Results of flexural rigidity mathematical approach.***

The flexural rigidities of the studied sections are estimated using Eq. (1) and compared with the calculated ones. The applied moments are taken as the yield moment. It is noticed that Eq. (1) overestimates the flexural rigidities with ratios ranged from 1.37 to 5.17 with 1.79 average value, 1.54 median, 1.75 mode value, 0.61 standard deviation, and 33% coefficient of variance.

8 CONCLUSIONS

Investigating the moment-curvature relationships of different rectangular reinforced concrete sections using iterative method and taking into account the non-linear behavior of concrete shows that:

1. The crack propagation concept has a great effect in the load-deformation characteristics of flexural members.
2. Crack propagation concept leads to different moment-curvature scheme from the classical ones.
3. The flexural rigidities estimated using the proposed formula and the proposed mathematical approach show good agreement with the calculated ones (using iterative method).

4. The concept of equivalent moment of inertia and concrete elastic modulus for calculating immediate deflections gives results that follow the classical scheme of moment-curvature relationship.

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